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WEIGHTLESSNESS AND ARTIFICIAL GRAVITATION

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ABSTRACT: This work involves theoretical evaluation of the possibility of achieving simulated weightlessness and simulated gravitation over long periods of time. Included is a mathematical definition of weightlessness. It is shown that the period of weightlessness achievable in an aircraft flying a parabolic curve is less than 2 minutes. The conditions of the creation of artificial gravitation in a satellite are defined and some of the problems associated with the dynamics of a solid body under the conditions of artificial gravitation are examined.

Determination of Weightlessness /155[1]

A body in alternating motion is in a state of weightlessness when the main vector of the surface forces acting on it are equal to zero.

Aerodynamic and reaction forces are the surface forces for any flying apparatus. They are equal to zero under the conditions of the motion of a spacecraft.

A body in the state of weightlessness may experience internal forces caused by surface forces. Air pressure in a spacecraft is often maintained close to atmospheric pressure. The body of a cosmonaut thus experiences internal forces of compression while the body of the spacecraft experiences forces of expansion, although, both are in a state of weightlessness. It is not the absence of internal forces that is characteristic of the conditions of weightlessness, but their constancy for all points of the body, i.e., the gradient of internal forces is equal to zero.

The determination of weightlessness described herein is valid for the conditions where the dimensions of the body are small in comparison with the distance to the center of gravitation.

During motion in the central Newtonian field of gravitation, alternating internal forces develop in a body. These forces are caused by the irregularity of the gravitational field. By considering a body, for simplicity, as a

[1] Numbers in the margin indicate pagination in the foreign text.

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uniform beam falling freely to the center of gravitation, it is easy to show that the greatest expanding forces will occur in the cross-section of the beam passing through the center of its mass. This is found from the expression

[redacted]

where l is the length of the beam,
R is the distance from its center to the center of gravitation,
Q is the force of attraction of the beam to the center of gravitation.

By assuming for a near-earth satellite /156

[redacted]

we find:

[redacted]

This value may be disregarded.

Artificial Weightlessness

~~From the definition of weightlessness given here we derive the method~~ of creating artificial weightlessness. If we maintain the condition on the aircraft under which the thrust of the engine $\overline{P}$ at any given moment is equal to the sum of the aerodynamic forces $\overline{R}$, the aircraft will be in a state of weightlessness and will move in a parabolic trajectory like a body tossed into a vacuum.

We will evaluate the duration of continuous weightlessness, assuming that an aircraft of altitude h attains maximum velocity v_{max} at an angel α to the horizon and travels in a parabolic trajectory. The time of flight at the altitude of maximum h will be:

[redacted]

For the value v_{max} = 680m/sec and α = 45° we obtain T_{max} = 98 sec. The time T_{max}, as we see, does not exceed two minutes.

The long term conditions of weightlessness can be created only during the flight of artificial earth satellites, i.e., directly under the actual conditions of cosmic flight.

Creation of Artificial Gravitation

Under the conditions of cosmic flight, artificial weight (gravitation) can be created, for all practical purposes, only by imparting a rotational motion to the spacecraft. If the cosmonaut is thus located at a distance a from the axis of rotation, the sensation of "normal weight" will correspond to the condition $a\omega^2 = g$. Assuming that acceptable working conditions can be provided for the cosmonaut with the partial restoration of weight, we will proceed from the dependence $a\omega^2 = \alpha g$, where $\alpha < 1$. Let us evaluate the energy balance for the creation of artificial gravitation.

Let Q be the weight of the spacecraft;
d is diameter;
ρ the radius of earth inertia relative to the axis of rotation;
a the distance of the cosmonaut from the axis of rotation.

A rotational moment M is created by two solid-fuel thrusters with a specific thrust of P_{sp}. When the thrusters are located at a distance of $\frac{d}{2}$ from the axis of rotation, we have the relations

[illegible]

hence /157

[illegible]

Assuming Q = 5000 kg, d = 5m, $\rho = 0.35d$, a = 2m, $\alpha = 0.5$, and P_{sp} = 200 sec, we find that q_{fe} = 4.8 kg.

Thus with the expenditure of 5 kg of fuel it is possible to create the condition of artificial gravity in a 5-ton spacecraft for the entire duration of the flight of the craft. If the rotation of the spacecraft has to be terminated prior to reentry it will be necessary to create the opposite rotational moment by burning the same amount of fuel, i.e., about 5 kg (for $\alpha = 1$, $q_{fe} \sim 10$ kg).

Certain Problems of Dynamics Under Conditions of Artificial Gravitation

In contrast to a virtually constant field of thrust at the surface of the earth, centrifugal forces in an artificial gravitational field increase linearly as the distance from the axis of rotation increases. Considering the possibility of using mechanical instruments aboard a rotating spacecraft it is advisable to consider certain features of the dynamics of solid fuel under the conditions of artificial gravitation. It is convenient to analyze these features by way of the example of motion of a solid body around a stationary point. It is necessary to exclude the forces of gravitation from the equations of motion, as applied to the counting system related to a revolving spacecraft, and to consider the centrifugal and Coriolis forces.

Let us first examine the spherical pendulum. To the spacecraft we will tie the right hand coordinate system $O\xi\eta\zeta$ (Fig. 1), where O is the point from which the pendulum is suspended, $O\zeta$ is parallel to the angular velocity vector $\bar{\omega}$ of the spacecraft, $\overrightarrow{\overline{OO}} = \bar{a}$ is a perpendicular extending from the origin to the vector $\bar{\omega}$. The direction of axis $\xi\eta$ is clear from the drawing. The position of material point M of the pendulum is defined by the vector radius $\bar{l}$:

[illegible]

or by angles α and β, which are indicated in the drawings. The distance of point M from the vector $\bar{\omega}$ will be

[illegible]

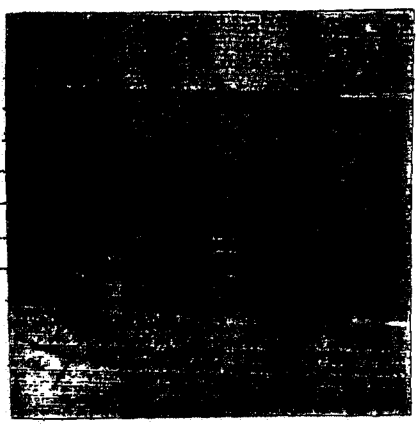

Figure 1.

The vector of the centrifugal force of inertia is

[illegible] (1)

and the moment of this force relative to the origin of the coordinates is

[illegible]

or finally,

[illegible] (2)

The force of Coriolis inertia will be:

[illegible]

or, after expanding, the double vector derivative /158

[illegible] (3)

The moment of this force relative to the origin is

[redacted]

or

[redacted]

After expanding the vector derivatives $\dot{\bar{\alpha}} \times \bar{l}$ and $\dot{\bar{\beta}} \times \bar{l}$, we have:

[redacted] (4)

It is clear from the last expression that $M_0^k = 0$ when the pendulum oscillates in plane $\xi O \eta$.

Thus, when studying the motion of the spherical pendulum, it is necessary to consider the moments $\bar{M}_0^c$ and $\bar{M}_0^k$ given by formulas (2) and (4).

Here,

[redacted]

We will notice that when the oscillations of the pendulum near the artificial vertical line $O\eta$ are short the values of the first order of smallness are α, β, $\dot{\alpha}$, $\dot{\beta}$, ξ, ζ. Here M_0^c is the value of the first order of smallness and M_0^k is the value of the second and highest order of smallness. By discarding the values of the order of smallness greater than the first and substituting

[redacted]

we find the equations for the oscillations of the spherical pendulum in the form

[redacted]

or finally,

[redacted] (5)

(6)

We will now consider the particular case of the oscillation of the spherical pendulum in the equitorial plane $\xi O \eta$, where $\beta = 0$, $\beta = 0$, and $\zeta = 0$. We obtain:

[redacted]

The pendulum completes its harmonic oscillations with the period:

[redacted]

If we select ω from the condition $a\omega^2 = g$, the period of oscillation of the pendulum under the conditions of artificial gravitation will coincide with the period of its oscillation at the surface of the earth:

[redacted]

When the point of suspension is located on the axis of rotation ($a = 0$), [redacted] The pendulum then does not complete its harmonic oscillations.

We will now examine the case where the pendulum oscillates in the axial plane $\zeta O \eta$.

Then $\alpha = 0, \dot{\alpha} = 0$, and $\xi = 0$. The equation for the oscillation of the pendulum becomes: /159

[redacted]

and

[redacted]

Thus the period of oscillation of the pendulum in the axial plane is less than that in the equitorial plane. The difference $T_{eq} - T_{ax}$ will decrease with a reduction in the ratio $\frac{1}{a}$.

We will now consider the case of the motion of a solid body around a stationary point, specifically, rotation at angles α and β (Fig. 2) where the body does not revolve around axis l_c.

[redacted]

Figure 2.

For the purpose of computing the elementary moments $d\overline{M}_0^c$ and $d\overline{M}_0^k$, we may use formulas (2) - (4). Let us turn our attention to the right hand coordinate system cxyz, which is related to the solid body (Fig. 2). We will place the origin of the coordinates at the center of the mass of the body and we will direct axis y along the vector $\overrightarrow{OC} = \overline{l}_c$ and axis x perpendicular to the plane of angle β such that $Ox \parallel O\xi$ for $\alpha = 0$. We will assume that axes cxyz are the principal axes of inertia. We will then express the coordinates ξ, η, and ζ through x, y, and z:

[redacted] (7)

From expression (2) we find:

[redacted]

By substituting the values of ξ, η, and ζ from (7), considering that for the principal and central axis of inertia

[redacted]

we find:

[redacted] (8)

From expression (4) we have for the moment of the Coriolis force:

[redacted]

After substituting the values of the coordinates ξ, η, and ζ from (7) we will have:

[redacted]

$$M_\eta^k = -2\omega\dot{\alpha}[m_c\zeta_c\xi_c + \sin\beta\cdot\cos\beta\cdot\sin\alpha\,(I_z - I_y)] - -2\omega\dot{\beta}[m_c\zeta_c^2 + I_0 - I_z\cos^2\beta - I_y\sin^2\beta]\cos\alpha,$$
$$M_\zeta^k = 2\omega\dot{\beta}m_c\zeta_c(\eta_c\cos\alpha + \xi_c\sin\alpha) + 2\omega\dot{\beta}\cdot\sin\beta\cdot\cos\beta\,(I_z - I_y). \qquad (9)$$ /160

Here $I_0 = \frac{1}{2}(I_x + I_y + I_z)$.

Finally, we will examine the particular case of the oscillation of the physical pendulum in the equitorial plane. In this case $\beta = 0$, $\dot{\beta} = 0$ and $\zeta_c = 0$,

$$M_\zeta^u = a\omega^2 m_c\xi_c = -a\omega^2 m_c l_c\sin\alpha,$$
$$M_\zeta^k = 0.$$

Equation for oscillation will have the form:

$$I_z\ddot{\alpha} = -a\omega^2 m_c l_c\sin\alpha.$$

However, in the case of small oscillations

$$\ddot{\alpha} + \frac{a\omega^2 m_c l_c}{I_z}\alpha = 0,$$

Hence

$$T_{ef} = \frac{2\pi}{\omega}\sqrt{\frac{I_z}{am_c l_c}},$$
$$g_{pr} = a\omega^2.$$

By analyzing the oscillations in the axial plane ($\alpha = 0$, $\dot{\alpha} = 0$, and $\xi_c = 0$) we find:

$$M_\zeta^c = -\omega^2[m_c\zeta_c(a + \eta_c) + (I_z - I_y)\sin\beta\cdot\cos\beta] =$$
$$= -\omega^2 m_c l_c\sin\beta\,(a + l_c\cos\beta) - \omega^2(I_z - I_y)\sin\beta\cdot\cos\beta,$$
$$M_\zeta^k = 0.$$

For small angles β the equation for the oscillations of the physical pendulum will be

$$\ddot{\beta} + \frac{\omega^2}{I_x}\left[m_c l_c (a + l_c) + I_z - I_y\right]\beta = 0,$$

hence,

$$T_{ax} = \frac{2\pi}{\omega}\sqrt{\frac{I_x}{m_c l_c (a + l_c) + I_z - I_y}},$$

$$g_{pr} = (a + l_c)\,\omega^2 + \frac{I_z - I_y}{m_c l_c}.$$

If the support of the physical pendulum passes through its mass center, then, in contrast to the oscillation of the pendulum in a constant gravitational field, in this case we have

$$M^c = -\omega^2 (I_z - I_y)\sin\beta\cdot\cos\beta \neq 0, \qquad (10)$$

i.e., the physical pendulum, suspended from the mass center, is in a state of invariable equilibrium only when $I_z = I_y$. This corresponds to the condition where the axis of rotation of the pendulum, which passes through its mass center, is its axis of symmetry. It is clear from expression (10) that when β = 0 stable equilibrium will occur only when $I_z > I_y$, i.e., the pendulum has a tendency to be determined by the long side along the artificial vertical On.

REFERENCE

1. Targ, S. M., Nevesomost', Weightlessness, Physical Encyclopedic Dictionary, 1963.

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